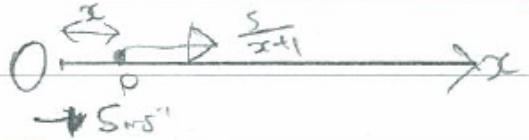
M3 - January 2002

1 -



$[F = ma]$

$$\frac{5}{x+1} = 0.2v\frac{dv}{dx}$$

$$5\int_0^x \frac{1}{x+1}\,dx = 0.2\int_5^{15} v\,dx$$

$$5\left[\ln|x+1|\right]_0^x = 0.2\left[\tfrac{1}{2}v^2\right]_5^{15}$$

$$50\ln|x+1| = 225 - 25$$

$$\ln|x+1| = 4$$

$$x+1 = e^4$$

$$x = e^4 - 1 = 53.6\text{m (3sf)}$$

2-

a)

b) $T = \frac{\lambda x}{a} = \frac{19.6 \times 2}{2}$

$= 19.6\,N$

$0.5 \times 9.8 \times (2+x) = \frac{\lambda x^2}{2a}$

$9.8 + 4.9x = 4.9x^2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ or -1

But $x > 0$ $\therefore x = 2$

$AC = 2 + x = 2 + 2 = 4m$

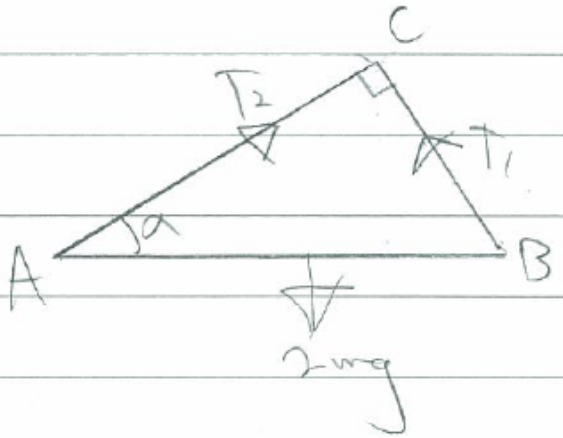
$\uparrow [F = ma]$

$19.6 - 4.9 = 0.5a$

$14.7 = 0.5a$

$a = 29.4\,ms^{-2}$

3-



a) $AC \neq AB$

b) $\uparrow 2mg = T_2 \sin\alpha + T_1 \cos\alpha$ ①

$\rightarrow T_2 \cos\alpha = T_1 \sin\alpha$

$T_2 = T_1 \tan\alpha$

$T_2 = \frac{3}{4} T_1$ ②

② in ①:

$2mg = \frac{3}{4} T_1 \sin\alpha + T_1 \cos\alpha$

$2mg = \frac{3}{4} \cdot \frac{3}{5} T_1 + \frac{4}{5} T_1$

$2mg = \left(\frac{9}{20} + \frac{4}{5}\right) T_1$

$T_1 = \frac{2mg \times 20}{25} = \frac{8mg}{5}$

$T_2 = \frac{3}{4} T_1$

$= \frac{3}{4} \times \frac{8mg}{5}$

$= \frac{6mg}{5}$

c) $T = \frac{\lambda x}{a}$

$\frac{8mg}{5} = \frac{kmgx}{a}$

$k = \frac{8a}{5x}$

$BC = 2a \times \frac{3}{5} = \frac{6a}{5}$

$x = \frac{6a}{5} - a = \frac{a}{5}$

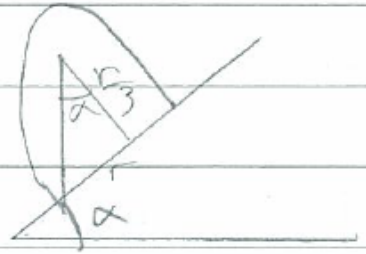
$k = \frac{8a}{5 \cdot \frac{a}{5}} = \frac{8a}{a} = 8$

4- a) $\int \pi y^2 x\,dx = \int_0^r \pi y^2\,dx \cdot \bar{x}$

$\pi \int_0^r x^2\,dx = \pi r \bar{x} \int_0^r x\,dx$

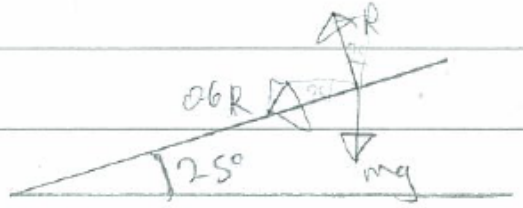
$\pi r \left[\frac{1}{3}x^3\right]_0^r = \pi r \bar{x} \left[\frac{1}{2}x^2\right]_0^r$

$\bar{x} = \frac{\frac{1}{3}r^3}{\frac{1}{2}r^2} = \frac{2}{3}r$

b) 

$$\tan\alpha = \frac{r}{\frac{r}{3}} = r \times \frac{3}{r} = 3$$

$$\alpha = 72° \ (2sf)$$

5. 

$\uparrow \quad mg + 0.6R\sin 25 = R\cos 25$

$mg = R\cos 25 - 0.6R\sin 25$

$mg = R(\cos 25 - 0.6\sin 25)$

$R = \dfrac{mg}{\cos 25 - 0.6\sin 25}$

$\leftarrow [F = ma]$

$R\sin 25 + 0.6R\cos 25 = \dfrac{mv^2}{40}$

$$\frac{mg\sin 25 + 0.6mg\cos 25}{\cos 25 - 0.6\sin 25} = \frac{mv^2}{40}$$

$$v^2 = \frac{40g\sin 25 + 0.6 \times 40g\cos 25}{\cos 25 - 0.6\sin 25} = 580$$

$v = 24 \text{ ms}^{-1} \ (2sf)$

6 -

a) $v^2 = \omega^2(a^2 - x^2)$, $a = 1.2$ If SHM about O

A + A

$\left(\frac{3}{10}\sqrt{3}\right)^2 = \omega^2\left(1.2^2 - 0.6^2\right)$

$\frac{27}{100} = \omega^2(1.08)$

$\omega^2 = \frac{1}{4}$

A + B

$\left(\frac{1}{5}\sqrt{5}\right)^2 = \omega^2\left(1.2^2 + 0.8^2\right)$

$\frac{1}{5} = \omega^2(0.8)$

$\omega^2 = \frac{1}{4}$

b) $v_{max} = \omega a = \frac{1}{2} \times 1.2 = 0.6\ ms^{-1}$

c) $a = \omega^2 x = \frac{1}{4} \times 0.6 = 0.15\ ms^{-2}$

d) $x = 1.2\sin\left(\frac{t}{2}\right)$

$0.6 = 1.2\sin\frac{t}{2}$

$\frac{t}{2} = \frac{\pi}{6}$

$t = \frac{\pi}{3}$

$0.8 = 1.2\sin\frac{t}{2}$

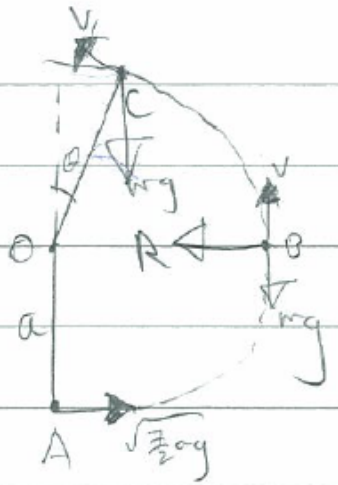
$\frac{2}{3} = \sin\frac{t}{2}$

$t = 1.46$

Time taken

$1.46 - \frac{\pi}{3} = 0.4(2.5)(3.$

7-



a) $ME_A = ME_B$

$\frac{1}{2}mv^2 = \frac{1}{2}m\frac{7}{2}ag = \frac{1}{2}mv^2 + mg.a$

$\frac{7}{2}ag = v^2 + 2ga$

$v^2 = \frac{3}{2}ag$

$\leftarrow$ [F = ma]

$R = \frac{mv^2}{r} = \frac{m \cdot \frac{3}{2}ag}{2a}$

$= \frac{3}{2}mg$

b) $ME_A = ME_C$

$\frac{1}{2}m\frac{7}{2}ag = \frac{1}{2}mV_1^2 + mg(a + a\cos\theta)$

$\frac{7}{2}ag = V_1^2 + 2ga + 2ga\cos\theta$

$V_1^2 = \frac{3}{2}ag - 2ag\cos\theta$

$\frac{2}{2}$

$[F=ma]$

$mg\cos\theta = \frac{m}{r}\left(\frac{3}{2}ag - 2ag\cos\theta\right)$

$g\cos\theta = \frac{3ag}{2a} - \frac{2ag\cos\theta}{a}$

$\cos\theta = \frac{3}{2} - 2\cos\theta$

$\cos\theta = \frac{1}{2}$

$\theta = 60^{\circ}$

c) 

$V_1^2 = \frac{3}{2}ag - 2ag\cos\theta = \frac{3}{2}ag - 2ag\frac{1}{2} = \frac{1}{2}ag$

Time taken $= \frac{a\sin\theta}{V_1\sin\theta} = \frac{a}{V_1} = \frac{a}{\sqrt{\frac{1}{2}ag}}$

$= \frac{a\sqrt{2}}{\sqrt{ag}} = \frac{a\sqrt{2ag}}{ag} = \frac{\sqrt{2ag}}{g}$